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SUMMARY

The hot-wire anemometer suggests itself as a promising method for measuring the fluctuating air velocities found in turbulent air flow. The only obstacle is the presence of a lag due to the limited energy input which makes even a fairly small wire incapable of following rapid fluctuations with accuracy. This paper gives the theory of the lag and describes an experimental arrangement for compensating for the lag for frequencies up to 100 or more per second when the amplitude of the fluctuation is not too great. An experimental test of the accuracy of compensation and some results obtained with the apparatus in a wind-tunnel air stream are described. While the apparatus is very bulky in its present form, it is believed possible to develop a more portable arrangement.

INTRODUCTION

In a historic paper (reference 1) Osborne Reynolds distinguished experimentally two types of fluid motion in pipes which are to-day termed laminar and turbulent. In the laminar motion the fluid particles follow each other in paths parallel to the axis of the pipe and the flow is steady—i. e., the speed at any point is not a function of the time. In the turbulent motion the fluid is filled with a mass of eddies, and while on the average the flow is parallel to the axis of the pipe there is a rapid fluctuation of the velocity at any point about a mean value. A stream of smoke or colored fluid introduced into a laminar flow retains its identity for a considerable time, whereas in a turbulent flow the stream is very rapidly diffused throughout the fluid. In laminar flow the stresses are transferred from layer to layer of the fluid solely by the action of molecular diffusion, whereas in turbulent flow there is in addition a molar diffusion or the transfer of momentum by extensive groups of molecules. The velocity distribution in laminar flow may be computed from the equations of motion of an incompressible viscous fluid on the assumption that a steady state of flow exists in which the lines of flow are parallel to the axis of the pipe. The speed is found to be a parabolic function of the radial distance from the axis of the pipe. In turbulent flow the speed is found to vary as a power of the distance from the wall of the pipe over most of the cross section, but it can not as yet be shown how this result may be derived from the equations of motion nor can a physical meaning be attributed to the exponent of the power law.

It was shown experimentally by Reynolds that turbulent motion could not exist if the value of the ratio $VD\rho/\mu$, now known as the Reynolds Number, was less than 2,000. V denotes the mean speed of flow, D the diameter of the pipe, ρ the density, and μ the viscosity of the fluid. At Reynolds Numbers much larger than 2,000 laminar motion is not possible, while in a certain intermediate range laminar motion is unstable, the degree of stability being dependent on the magnitude of the disturbances present.

These conceptions of laminar and turbulent motions and of the Reynolds Number as a criterion determining the type of flow have been extended to fluid motion in general. In the extension the designation "turbulent motion" has sometimes been restricted to those cases in which the fluctuations of velocity are more or less random in nature. In general, however, the periodic motions which occur behind many bodies as, for example, behind cylinders, and which may be approximately represented as the effect of a limited number of moving singularities (vortices), are also classed as turbulent motions.

In wind tunnels it is desired to reproduce as closely as possible the same relative flow as if the model were moving forward in still air. To obtain this result, it is necessary to secure an air stream of large cross section as compared to the cross section of the model and of uniform and steady speed throughout. It is at once apparent that the ideal condition can not be secured. In the first place the Reynolds Numbers required for even moderate speeds and cross-sectional areas are far above 2,000, so that the flow is of necessity turbulent. In the second place, the distribution of average speed in fully developed laminar or turbulent flow (i. e., far from the entrance of a tube) is not uniform across the section and therefore unsatisfactory. The compromise usually made is to work near enough to the entrance of a large tube that the effect of the walls does not have time to reach the center. Conditions are therefore not steady, and turbulence is always present.

Turbulence in wind tunnels does not seem to produce any large effects as judged by a comparison of results obtained on similar models in different wind tunnels except for cylinders, spheres, spheroids, ellipsoids, and airship hulls. In these instances very large effects are produced. (References 2 and 7.) The desire to make some direct measurement of the turbulence in wind tunnels was the incentive for the work here described, but before proceeding further it is desirable to specify in more detail what is meant by the word "turbulence."

In 1895 Reynolds (reference 3) gave a mathematical formulation of turbulent flow. He began by a consideration of the meaning of the velocities entering into the general equations of motion. The equations are obtained on the assumption of a continuous medium whereas we know in fact that the medium is discontinuous, consisting of individual molecules. We think of the velocity at any point at a given instant not as the velocity of a single molecule at that point but as the velocity of a fictitious fluid particle. We imagine a small volume around the point, for example, a small sphere of radius r, and define the velocity at the point V, as the average velocity of all the molecules within the sphere. To satisfy the conditions of continuity, the sphere must be so small that the variation of average velocity taken about other points within the sphere is small. Thus if we consider the variation in any direction s, $r\frac{d^2V}{ds^2}$ must be negligible in comparison with $\frac{dV}{ds}$. On the other hand, the sphere must be large compared with the mean free path of the molecules. After an infinitesimal interval of time dt, we assume that the "particle" has reached the point Vdt. The fluid particle may thus be given a definite mathematical meaning, but it has no physical existence and in particular it does not consist always of the same molecules.

It is possible to define the velocity at a point in a manner not involving the conception of a fluid particle by taking the vectorial average velocity of all molecules passing through a small volume around the point in a certain time dt, which is long enough to include a large number of molecules.

The effect of the motion of the molecules is taken care of in the theory by a system of stresses, which are explained as due to the transfer of momentum by the molecular motions. In terms of the kinetic theory of gases for the simple case of two-dimensional parallel flow, the transfer of momentum per unit area per unit time is proportional to the density, the mean molecular speed, the mean free path, and the velocity gradient. In terms of hydrodynamical theory, the stress is equal to the product of the coefficient of viscosity, which is a constant for a given fluid at a given temperature, and the velocity gradient.

Reynolds proposed to treat the problem of turbulence in somewhat the same way. He proposed to substitute for the real motion an average or mean motion on which fluctuations were superposed and to take account of the effect of the fluctuations by a system of stresses. In the simple case of two-dimensional parallel flow the additional shearing stress turns out to be $\rho\,\overline{uv}$ times the velocity gradient where ρ is the density and $\overline{uv}$ is the mean value of the product of the superposed fluctuating components parallel and at right angles to the mean flow. Reynolds was also able to set up equations giving the relations which must exist between the variables in the steady state (in the kinetic sense)—namely, the condition that the rate at which the energy of the mean motion is converted to energy of the fluctuations is equal to the rate at which the energy of the fluctuations is dissipated by viscosity or in terms of the kinetic theory converted into the energy of the molecular fluctuations or heat motion.

H. A. Lorentz (reference 4) and J. M. Burgers (reference 5) have stated the equations of Reynolds in a somewhat different form and have shown that the knowledge of three mean values is sufficient to permit the computation of the resistance. These are the mean value of the product $\overline{uv}$, the mean value of $\overline{uv^2}$, and the mean value of the square of the vorticity.

Th. von Karman (reference 6) has given an excellent summary of the theory of turbulence developed along these lines to which the reader is referred for a summary of the mathematical treatment of the subject. It is clear from this summary that the treatment outlined by Reynolds has not as yet led to anything conclusive, principally because there is no method of computing the required mean values and because there is not sufficient experimental data on the fluctuations to give a clue to fruitful hypotheses. The great need at the present time in the further development of the theory of turbulence is more experiments on the actual fluctuations to supplement the data already available on the distribution of mean velocity.

It has been pointed out that the ordinary theory of laminar flow is based on certain average velocities which could be defined in terms of space averages or in terms of time averages. The averages taken for turbulent flow to obtain the mean or fundamental motion may also be based on either space or time averages, and it is usually assumed that the same result will be obtained in all cases provided the spaces and times over which the averages are taken are suitably chosen. It is clear that this method of procedure can be applied to a given flow in a great many ways by using space and time intervals of different orders of magnitude. The result obtained depends on what we wish to consider as the basic or fundamental flow. The process is an artificial one based on convenience, and some of the confusion as to the meaning of turbulence arises from the failure to appreciate this fact. For example, in the case of the natural wind we might consider averages over 5-minute intervals, 5-second intervals, or 0.05-second intervals, depending on the purpose in question. In no case would the fundamental motion be an absolutely stationary one, and in every case the conception of turbulence would be different. In most practical cases we define the fundamental flow as the one indicated by the instruments at our disposal which give mean values over a period ranging from, say, one second to one minute. Every physical instrument gives some kind of an average over a certain volume determined by its size and over a certain interval of time determined by its inertia or other lag characteristics. With the above definition of the fundamental motion we may have as many definitions of turbulence as we have instruments at our disposal.

The particular conception of turbulence adopted in this paper is based on a fundamental motion defined by time averages taken over several seconds. Any variation with frequency greater than one per two or three seconds is included in the superposed turbulence. For the present we have no means of taking averages over any extensive volume and no means of obtaining the particular average values used in the present form of the theory. It is the object of this paper to outline a method by means of which the fluctuations of the air speed at a given point may be measured and to give typical results obtained in the turbulent flow in a wind tunnel.

PREVIOUS WORK AT THE BUREAU OF STANDARDS

In Technical Report No. 231 of the National Advisory Committee for Aeronautics (reference 7), the first work carried out at the Bureau of Standards on the problem of turbulence in wind tunnels is described. One section of that report deals with measurements of the air resistance of cylinders in the turbulent flow behind wire screens of varying mesh, and another section deals with attempts to measure fluctuations of static and impact pressures by means of a diaphragm pressure gauge connected to the pressure nozzle by rubber tubing.

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In practical wind-tunnel measurements we are interested in the effects of turbulence on the air forces on the models under investigation, but experiment shows that the results obtained for one model can not be applied to another. This seemingly direct method of attack also suffers from the difficulty that no numerical value can be assigned to the turbulence produced by the wire screens commonly used and that there is no method of extrapolation to obtain the result for an air stream free from turbulence.

Attention has therefore been turned to the more fundamental problem of measuring directly the variation of static pressure, speed, and direction of the air stream. Since the publication of National Advisory Committee for Aeronautics Technical Report No. 231 (reference 7), much attention has been given to the development of methods involving the transmission of pressure and to the possible study of directional changes by means of small, light, freely pivoted vanes. All of these mechanical methods have been unsuccessful, primarily due to the large inertia or the low natural frequency of the mechanical system. All such systems seem to oscillate with their natural frequency, abstracting the required small energy from the air stream.

In February, 1926, we began an investigation of the possibility of using a hot-wire anemometer for the measurement of speed fluctuations. After several months' work with a wire about 0.075 mm in diameter, it was concluded that the lag in the heating and cooling of the wire was so great that the method was not very promising. At about this time a paper by Prof. J. M. Burgers, of Delft, on Experiments on the Fluctuations of the Velocity in a Current of Air (reference 10) was called to our attention, in which experiments with wires of much smaller diameter than we had used gave very promising results. We were, however, unable to resume work on the problem until July, 1927. During the summer of 1927 we were fortunate in having associated with our staff Dr. Arthur E. Ruark for a period of two and one-half months. Doctor Ruark assembled a great deal of the necessary electrical apparatus, instructed other members of the section in its use, and made a number of experiments with several suggested arrangements. While Doctor Ruark was not associated with us in the design of the circuits finally used, it is a pleasure to acknowledge his contribution.

INVESTIGATIONS WITH THE HOT-WIRE ANEMOMETER

The classical work on the hot-wire anemometer as an instrument for measuring air speed is that by L. V. King. (Reference 8.) Many additional papers on the subject have appeared since that paper was published, many of them dealing with various modifications of the electrical circuits and of the form of the wire mounting. It is not proposed to give any complete bibliography, and we shall, in fact, refer only to those experiments in which measurements of fluctuations were attempted. The first work of this kind with which we are familiar is that by E. Huguenard, A. Magnan, and A. Planiol (reference 9) on the measurement of gusts in natural winds. These investigators give a brief theoretical treatment of the problem of the lag of the hot-wire anemometer and a method of computing corrections for lag.

Two years later the paper of Professor Burgers (reference 10), which has already been referred to, was published. Burgers also gave a brief computation of the lag, and by the use of wires about 0.015 mm in diameter he was able to reduce the lag considerably. He showed by the use of two instruments that the wire should not be longer than about 1 cm, since this was the maximum spacing for which the two wires gave the same indications at a given instant. Burgers described a method of recording directional variations.

In 1927 A. Fage and F. C. Johansen (reference 11), of the National Physical Laboratory, described some measurements of fluctuations behind plates in connection with measurements of average speed and direction. A wire 0.025 mm in diameter was used in conjunction with a string galvanometer and the measurements were made at low wind speeds to minimize the effect of lag.

The papers of Anrep, Downing, and their coworkers (reference 12) deserve mention in this connection, since the method of computing the correction for lag given by Huguenard and his coworkers was independently derived.

COMPUTATION OF THE LAG OF A HOT-WIRE ANEMOMETER

Since the theoretical treatment of the lag of the hot-wire anemometer given in the papers referred to above does not lend itself readily to the computation of the response of the instrument to an irregular or even to a periodic fluctuation of speed, we wish to give a more comprehensive computation:

The following symbols will be used:

$\frac{dH}{dt}$ = rate of increase of heat energy in the wire.
i = heating current (maintained constant).
R = instantaneous resistance of the wire.
T = instantaneous temperature of the wire.
$\overline{R}$ = average resistance of the wire.
$\overline{T}$ = average temperature of the wire.
T_o = air temperature (room temperature).
R_o = resistance of the wire at temperature T_o.
V = instantaneous air speed.
R_e = resistance of the wire in an air stream of constant speed V.
T_e = temperature of the wire in an air stream of constant speed V.
m = mass of the wire.
s = specific heat of the wire.
α = temperature coefficient of resistance of the wire.
$\overline{V}$ = average wind speed.
p = 2π times frequency of air speed variation.
$K(T-T_o)$ = rate of heat loss from wire by radiation and free convection.
$C(T-T_o)$ = rate of heat loss from wire by forced convection of air stream of speed V.

We assume that the rate of heat loss from the hot wire in an air stream of speed V is given by King's equation (reference 13), rate of heat loss equals $K(T-T_o)+C(T-T_o)\sqrt{V}$, in other words, that the rate of heat loss does not depend on the rate of variation of the air speed. We assume further that the heating current i, is maintained at a constant value and we understand by T the mean temperature at any instant as determined by the instantaneous resistance of the wire. By expressing the fact that the rate at which heat energy accumulates in the wire is equal to the rate at which electrical energy enters, less the rate at which heat energy leaves, we obtain the fundamental equation

$$\frac{dH}{dt}=i^2R-(K+C\sqrt{V})\,(T-T_o) \tag{1}$$

Now $\frac{dH}{dt}=4.2\,ms\,\frac{dT}{dt}$ since the increase in heat energy produces an increase in the temperature of the wire. Likewise $T-T_o=\frac{R-R_o}{R_o\alpha}$ and thus $\frac{dT}{dt}=\frac{1}{R_o\alpha}\frac{dR}{dt}$. On substitution of these values, we obtain

$$\frac{4.2ms}{R_o\alpha}\frac{dR}{dt}=i^2R-(K+C\sqrt{V})\frac{(R-R_o)}{R_o\alpha} \tag{2}$$

from which R is to be determined. If the cycle is performed very slowly so that $\frac{dR}{dt}=0$, the equilibrium value R_e, would be determined by

$$i^2R_e-(K+C\sqrt{V})\frac{(R_e-R_o)}{R_o\alpha}=0 \tag{3}$$

Equation (3) is the equation of the usual calibration curve of the wire and is found to be in accordance with the experimental results as to the dependence of R_e on i and V as shown in

Figures 1 and 2. The following discussion requires only that the heat loss be proportional to $f(V)(T-T_o)$ whether the wire be in temperature equilibrium or not. Solving equation (3) for $K+C\sqrt{V}$ and substituting in equation (2) we obtain

$$\frac{4.2ms}{R_o\alpha}\frac{dR}{dt}=\frac{i^2R_o(R_e-R)}{R_e-R_o} \tag{4}$$

and then by adding and subtracting R_o within the parenthesis on the right and some simple transformations

$$R_e-R_o=\frac{R-R_o}{1-\frac{4.2ms}{i^2R_o^2\alpha}\frac{dR}{dt}} \tag{5}$$

For simplicity in further discussion we rewrite this equation in the form

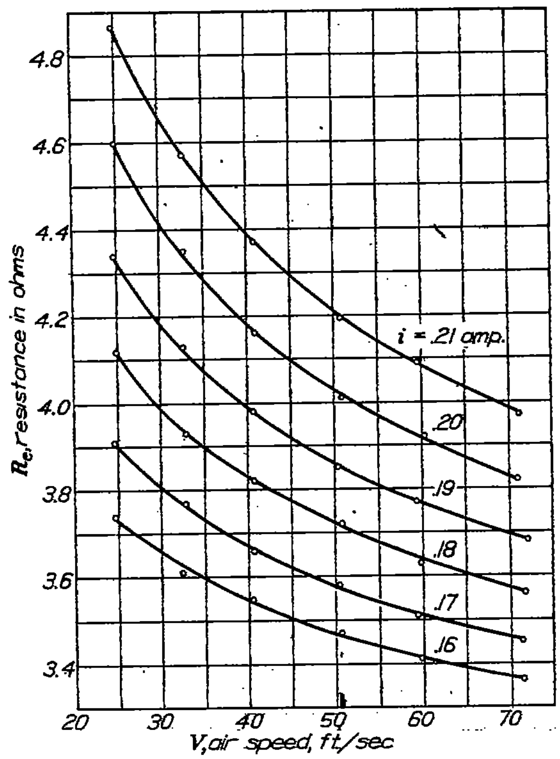


FIGURE 1.—Calibration curves for a hot wire (diameter, 17μ) at different heating currents

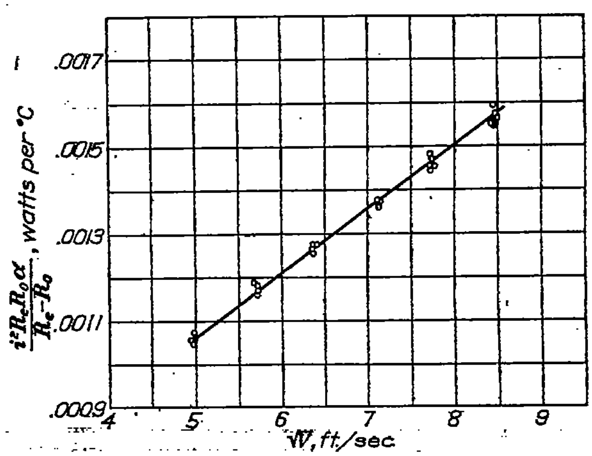


FIGURE 2.—The data on Figure 1 plotted to show the concordance with King's equation

$$\frac{R_e-R_o}{\overline{R}-R_o}=\frac{\frac{R-R_o}{\overline{R}-R_o}}{1-M\frac{d}{dt}\left(\frac{R-R_o}{\overline{R}-R_o}\right)} \tag{6}$$

where

$$M=\frac{4.2ms(\overline{R}-R_o)}{i^2R_o^2\alpha}=\frac{4.2ms(\overline{T}-T_o)}{i^2R_o} \tag{7}$$

M has the dimensions of time and is the single constant[1] necessary to characterize the behavior of the wire for a given heating current and operating temperature.

The physical meaning of M may be seen by supposing that the air speed is suddenly changed, so that R_e changes from a constant value of, say, R_1 to the value of $\overline{R}$, and investigating the change

[1] The variations of s and α with temperature are negligible under the conditions of operation.

of R. We find rather simply that $R-\overline{R}=(R_i-\overline{R})e^{\frac{-t}{M}}$. M is therefore the time required for $R-\overline{R}$ to become equal to $\frac{1}{e}$ times the original difference $R_i-\overline{R}$.[2]

It should be noted that the occurrence of the lag is not dependent on a thermal lag. When the speed decreases rapidly, the electric current is not able to supply sufficient energy to raise the temperature of the wire fast enough to correspond to the equilibrium temperature, and when the speed increases rapidly the supply of energy reduces the rate of cooling. The apparently unsymmetrical response obtained by Richardson (reference 14) was produced by oscillating the wire in its own convection current with approximately the same maximum speed as that of the convection current. The phenomena involved in a hot wire oscillated in still air are so complicated by the presence of the convection current and by the air motions set up by the wire and its mounting that this special case is understood to be excluded from the treatment given in this paper.

Equation (6) may be written

$$\frac{d}{dt}\left(\frac{R-R_o}{\overline{R}-R_o}\right)+\frac{\dfrac{R-R_o}{\overline{R}-R_o}}{M\left(\dfrac{R_e-R_o}{\overline{R}-R_o}\right)}=\frac{1}{M} \tag{8}$$

The formal solution of this linear equation when M may be regarded as constant is

$$\frac{R-R_o}{\overline{R}-R_o}e^{\int\frac{dt}{M\left(\frac{R_e-R_o}{\overline{R}-R_o}\right)}}=\int\frac{1}{M}e^{\int\frac{dt}{M\left(\frac{R_e-R_o}{\overline{R}-R_o}\right)}}dt+\text{const.} \tag{9}$$

in which, of course, R_e is a function of t determined from the velocity variation by means of equation (3). Unfortunately the integrations involved under any reasonable assumption as to the variation of velocity with time are very troublesome. Their evaluation in any useful form is beyond the skill of the authors. We therefore turn to simpler methods of attack.

It is obvious that it is always possible to reverse the problem, that is, knowing R and $\frac{dR}{dt}$, to determine R_e. This is the usual experimental problem. Let us suppose that we obtain on the record a sinusoidal curve represented by

$$R-R_o=(\overline{R}-R_o)\ (1+a\sin pt) \tag{10}$$

By substitution in (6) and simple transformations we obtain

$$\frac{R_e-R_o}{\overline{R}-R_o}=1+\frac{a\sqrt{1+M^2p^2}\sin\ (pt+\tan^{-1}Mp)}{1-apM\cos pt} \tag{11}$$

If apM is small compared to unity, R_e-R_o also undergoes a sinusoidal variation. We shall see later that in many cases apM is small, which means physically that the variations of resistance are small as compared to $\overline{R}-R_o$, hence the variations of temperature small as compared to the temperature head $\overline{T}-T_o$, and hence the speed variations small as compared to the mean speed. It is of interest therefore to investigate the second approximation and to see how large the speed variations may be. Expanding $\frac{1}{1-apM\cos pt}$ and performing the multiplication, we get

$$\frac{R_e-R_o}{\overline{R}-R_o}=1+a\sin pt+apM\cos pt+a^2pM\sin pt\cos pt+a^2p^2M^2\cos^2 pt+\ .\ .\ .\ . \tag{12}$$

[2] This computation was made by Huguenard and his coworkers, by Burgers, and by Anrep and his coworkers in the papers previously referred to.

Hence the average value of

$$\frac{R_e-R_o}{\overline{R}-R_o} \text{ is } 1+\tfrac{1}{2}a^2p^2M^2+\ \ldots\ . \tag{13}$$

A more complete calculation gives $\dfrac{1}{\sqrt{1-a^2p^2M^2}}$ for the average value of $\dfrac{R_e-R_o}{\overline{R}-R_o}$. Therefore if the resistance variations are large, the observed mean resistance is too low and the corresponding mean speed too high. This error in the mean speed is offset to some extent by the curvature of the resistance-speed curve which causes the mean resistance to correspond to a speed lower than the mean speed. apM may be as large as 0.14 without introducing an error in the mean $\dfrac{R-R_o}{\overline{R}-R_o}$ larger than 1 per cent, which means that the speed may vary by ± 15 to ± 20 per cent without introducing serious error in the mean speed on account of lag. The effect of curvature of the resistance-speed curve must, however, be considered for such large speed changes.

Similarly, by computing the maximum and minimum value of R_e-R_o we find for the approximate amplitude of the fluctuation in $\dfrac{R_e-R_o}{\overline{R}-R_o}$

$$\frac{a\sqrt{1+M^2p^2-a^2p^2M^2}}{1-a^2p^2M^2} \tag{14}$$

and the error in using $a\sqrt{1+M^2p^2}$ is less than 2 per cent for $apM=0.14$.

We now return to the case in which apM is neglected for more detailed consideration. Returning to equation (4) we suppose that R_e-R_o in the denominator on the right may be replaced by $\overline{R}-R_o$ without sensible error—i. e., that $R_e-\overline{R}$ is negligible compared to $\overline{R}-R_o$. We then obtain

$$M\frac{\mathrm{d}R}{\mathrm{d}t}=R_e-R \tag{15}$$

The solution of this linear equation consists of a transient term containing the factor $e^{-\frac{t}{M}}$ and a periodic term, if R_e is periodic. The transient term soon becomes negligible and will not be considered further. To obtain the periodic term, we suppose that R_e is expanded in a Fourier series such that

$$\begin{aligned} R_e-\overline{R}=a_1 \sin pt+a_2 \sin 2pt+\cdots+a_n \sin npt+\cdots \\ +b_1 \cos pt+b_2 \cos 2pt+\cdots+b_n \cos npt+\cdots \end{aligned} \tag{16}$$

We then assume that

$$\begin{aligned} R-\overline{R}=c_1 \sin pt+c_2 \sin 2pt+\cdots+c_n \sin npt+\cdots \\ +\mathrm{d}_1 \cos pt+\mathrm{d}_2 \cos 2pt+\cdots+\mathrm{d}_n \cos npt+\cdots \end{aligned} \tag{17}$$

whence

$$\begin{aligned} \frac{\mathrm{d}R}{\mathrm{d}t}=c_1 p \cos pt+2c_2p \cos 2pt+\cdots+nc_np \cos npt+\cdots \\ -\mathrm{d}_1 p \sin pt-2\mathrm{d}_2p \sin 2pt-\cdots-n\mathrm{d}_np \sin npt-\cdots \end{aligned} \tag{18}$$

We then have on substituting in (15)

$$\begin{aligned} Mnc_np \cos npt=b_n \cos npt-\mathrm{d}_n \cos npt \\ -Mn\mathrm{d}_np \sin npt=a_n \sin npt-c_n \sin npt \end{aligned} \tag{19}$$

whence

$$\mathrm{d}_n=\frac{b_n-a_n\ Mnp}{1+M^2n^2p^2},\ c_n=\frac{a_n+Mnp\ b_n}{1+M^2n^2p^2} \tag{20}$$

Thus

$$R-\bar{R}=\Sigma\left\{\frac{a_n}{1+M^2n^2p^2}\sin npt-\frac{a_n\,Mnp}{1+M^2n^2p^2}\cos npt+\frac{b_n}{1+M^2n^2p^2}\cos npt+\frac{Mnp\,b_n}{1+M^2n^2p^2}\sin npt\right\} \tag{21}$$

$$=\Sigma\left\{\frac{a_n}{\sqrt{1+M^2n^2p^2}}\sin\,(npt-\tan^{-1}Mnp)+\frac{b_n}{\sqrt{1+M^2n^2p^2}}\cos\,(npt-\tan^{-1}Mnp)\right\} \tag{22}$$

Hence the general result is that the term of frequency $\frac{np}{2\pi}$ has its amplitude diminished in the ratio $\frac{1}{\sqrt{1+M^2p^2n^2}}$ and suffers a phase retardation of $\tan^{-1} Mnp$. This is the generalization of equation (11) when apM is small.

We may state this important result as follows: If the equilibrium value of the resistance is expanded in a Fourier series, the actual value of the resistance will be such that the nth harmonic is reduced in amplitude in the ratio $\frac{1}{\sqrt{1+n^2p^2M^2}}$ and is retarded in phase by an amount $\tan^{-1} npM$ where p is equal to 2π times the fundamental frequency and M equals $\frac{4.2\,ms\,(\bar{T}-T_a)}{i^2\,R_a}$

Consider the case of platinum wire and let r be the resistivity at temperature T_a, l the length of the wire, A the area of cross section, and ρ the density. Then

$$M=\frac{4.2\rho lAs(\bar{T}-T_a)}{i^2r\dfrac{l}{A}}=\frac{4.2\rho A^2s(\bar{T}-T_a)}{i^2r}$$

For platinum, $r=0.000012$, $\rho=21.37$, $s=0.035$ approximately, and if operated at say 500° C., $M=1.31\times10^8\,\frac{A^2}{i^2}$. For wire 0.017 mm. in diameter, $A=2.265\times10^{-6}$ and taking i as 0.2 ampere, $M=0.0168$ second. For this time constant the values of the amplitude reduction factor and the phase shift are as follows:

Frequency	p	$\frac{1}{\sqrt{1+p^2M^2}}$	$\tan^{-1} Mp$
1	6.3	0.995	6°
5	31.4	.884	28°
10	62.8	.687	47°
20	135.7	.402	66°
50	314.2	.186	79°
75	471.2	.125	83°
100	628.3	.094	85°
200	1,256.6	.043	89°

The limitation of the hot wire for high frequencies is apparent. We see that while Burgers, for example, was able to obtain records of variations of reasonable frequency, the amplitudes were not properly reproduced. The value of M is found to vary roughly as the area of the wire (not as A^2 since i must be varied roughly as $\sqrt{A}$), and for a given wire and fixed temperature head to decrease as the wind speed increases (since i can be increased). It is advantageous to use as low a temperature as the sensitivity of the apparatus permits and we have in most cases used temperatures only 100° C. above the air temperature.

Returning to equation (15) the correction for lag may be made graphically by a simple construction. At a point P on the curve of the observed resistance R (fig. 3), draw the tangent and the perpendicular to the time axis. The value of R_e is given by the point on the perpendicular which is distant by an amount M measured parallel to the time axis from the tangent. A correction similar to this is given by Anrep and Downing (reference 15) as based on a method given by G. J. Burch (reference 16) for correcting for the lag of a capillary electrometer. It should be noted that the method is valid only if the speed variations are small. If the speed variations are large, a method along similar but not simple lines may be derived from equation (6).

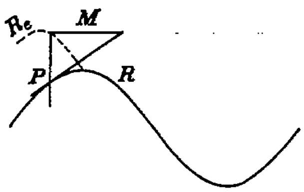


FIGURE 3.—Graphical method of correcting for lag

The form of equation (22) suggests that it might be possible to restore the amplitude and compensate for the lag of the hot wire by an electrical method. The equation is of the same form as that for the current in a circuit containing resistance R and inductance L, provided $\frac{L}{R}=M$, the time constant of the hot wire. If in an amplifier we can pass on to the succeeding stage the voltage variation in a circuit with this time constant, the amplitude can be restored, and an advancement of phase made to compensate for the lag of the wire. The development of such an arrangement is not as simple as might appear and we shall reserve until a later section a description of the compensating circuit, the theory of its action, and a description of the experimental test of its action. The compensation can not be made perfect as is obvious from the limiting case of high-frequency variations of air speed where the change of resistance of the wire approaches zero. We can not expect to get a positive result from a zero effect and the compensation is therefore confined to frequencies less than a certain upper limit.

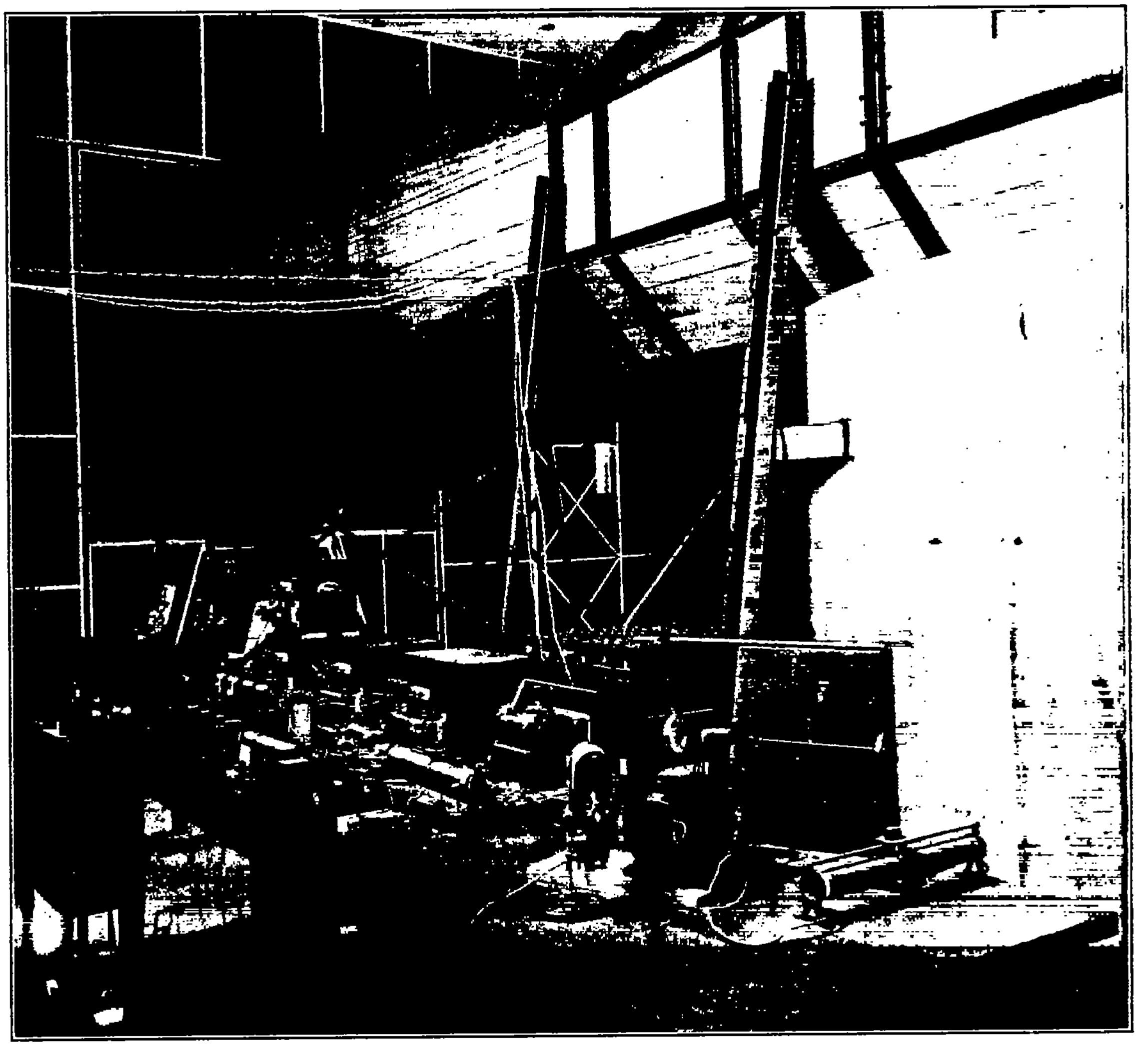


FIGURE 4.—General view of apparatus

GENERAL ARRANGEMENT OF APPARATUS

In the apparatus developed at the Bureau of Standards, the wire, of platinum about 0.017 mm in diameter, is heated from a 120-volt storage-battery line through a large swamping resistance so as to keep the heating current very nearly constant. Potential leads are provided for the measurement of the voltage drop across the wire, from which the resistance is readily computed. The resistance of the wire is a function of the air speed. Variations of air speed produce variations of resistance and hence of the voltage drop across the wire. The voltage

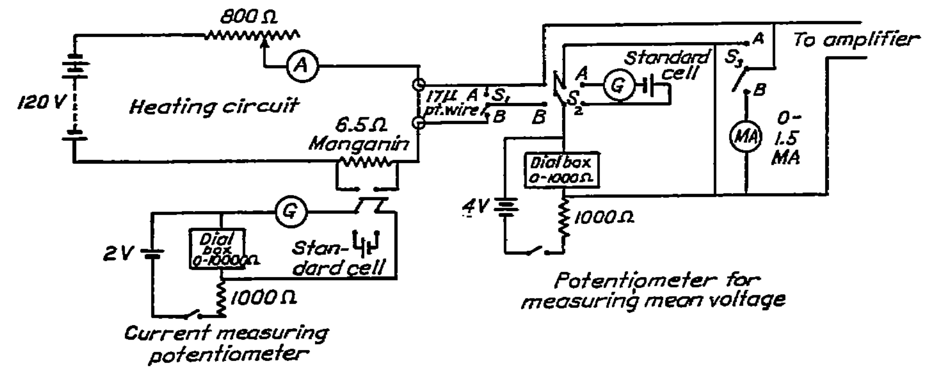


FIGURE 5A.—Wiring diagram of heating circuit and circuit for measurement of mean speed

variations are amplified by means of a direct current resistance coupled amplifier. In the amplifier is incorporated the compensating circuit already referred to, which compensates for the lag of the wire over a certain range of frequencies. The plate current of the last stage of the amplifier is passed through a fixed resistance and the voltage drop across this resistance is balanced by a voltage divider. In the balancing circuit there is placed, in addition to the direct-current milliammeter which serves as the indicating instrument of the voltage divider, an alternating-current milliammeter for measuring the alternating current produced by the fluctuations. If desired, an oscillograph element may be substituted for the alternating-current

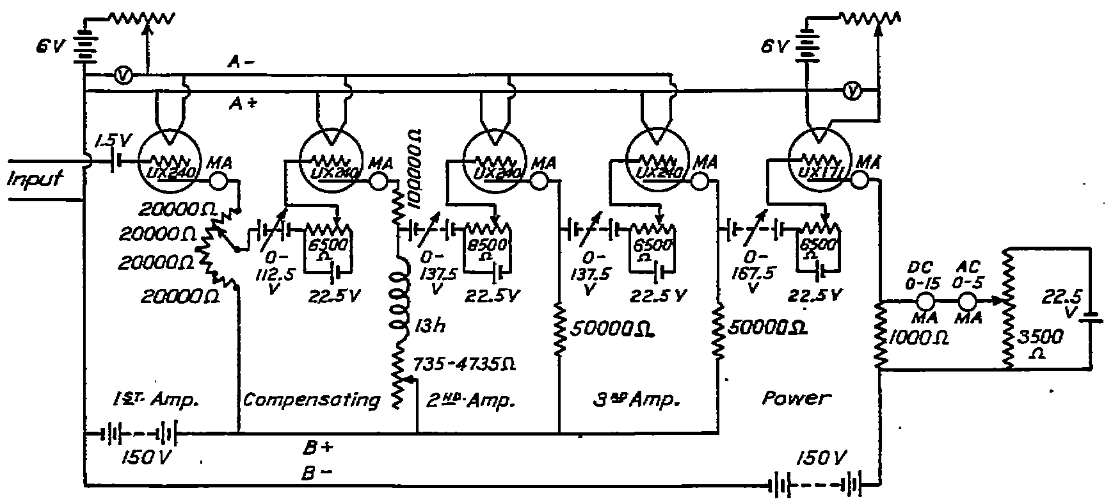


FIGURE 5B.—Wiring diagram of amplifier, compensator, and output circuit

milliammeter and the wave form photographed. A photograph of the apparatus is shown in Figure 4 and a wiring diagram in Figures 5a and 5b. For simplicity, certain protective switches and the oscillograph circuit are omitted in the wiring diagram.

THE HEATING CIRCUIT

It is essential to maintain the heating current reasonably constant. The electron tube used in the amplifier is essentially an electrostatic device and is operated by voltage changes. If one connects a hot wire directly to a battery, the voltage across the wire will remain approximately constant and equal to the battery voltage (absolutely equal except for the drop in the leads and in the battery) irrespective of changes in the resistance of the wire. Hence, to secure the greatest sensitivity it is desirable to maintain the current constant. The resistance of the wire when hot was usually about 4 ohms and the heating current was usually 0.2 ampere. The maximum variations of resistance in which we are interested do not usually exceed ±0.25 ohm. Thus by using the laboratory 120-volt storage-battery line with 596 ohms in series with the wire, the maximum change in the heating current during the fluctuation was about 1 part in 2,500, and a voltage fluctuation equal to 0.2, the value of the current, times the resistance fluctuations was impressed on the amplifier.

To measure the current accurately and to maintain it at a constant value as the mean resistance of the wire varied with changing mean speeds, a 6.5-ohm manganin resistance was placed in series with the wire, and the potential drop across this resistance was measured by an improvised potentiometer consisting of a 2-volt storage cell, a 10,000-ohm dial box, a standard 1,000-ohm resistance, a portable galvanometer, and a standard cell.

THE CIRCUIT FOR MEASURING MEAN RESISTANCE

The mean voltage drop across the wire was measured by a similar potentiometer except that a 4-volt storage cell, a 1,000-ohm dial box, and a milliammeter (range 0–1.5 milliamperes) were used. Switch S_2 in the diagram (fig. 5a) permits a check of the potentiometer by a standard cell. Switch S_1 enables the wire to be removed from the amplifier circuit, still retaining a closed circuit when the switch is closed in position A. Switch S_3 enables the removal of the indicating milliammeter from the circuit, retaining a closed circuit on the grid of the first amplifying tube when closed in position A. The procedure is as follows: S_2 is closed in position A and the potentiometer checked against the standard cell, S_2 is then thrown to position B and remains there subsequently. S_1 and S_3 are closed in position B and the potentiometer adjusted until the milliammeter fluctuates equally about the zero mark. By opening S_3 the fluctuations are passed on to the amplifier. After a reading has been taken, S_3 is closed in position A, S_1 thrown to position A, the dial box set to zero, and S_3 opened, the object of this procedure being to avoid disturbance to the amplifier by keeping the same average voltage on the grid of the first tube. After these adjustments, known voltages can be applied to the amplifier by turning the dial box, and the amplification can be measured.

THE AMPLIFIER

The amplifying circuit is one given by R. W. King (reference 17) as suitable for direct current amplification. No condensers or transformers are used and there is therefore no selective amplification. The coupling condensers of the more common resistance condenser coupled amplifier are replaced by large C batteries in series with a voltage divider for fine adjustment of the bias voltage. Since the reduction of the wiring diagram given by King to a workable amplifier was attended by some difficulties, it is desired to make a record of the more important ones for the benefit of others intending to use this type of direct-current amplifier. The wiring diagram given in Figure 5b gives the type of tubes used, the values of the battery voltages, and the coupling resistances.

The first difficulty encountered was that due to rapid drifting of the current in the last stage, caused partly by leakage currents from the C batteries, which are at high potential, and partly by small changes in voltage of the various batteries. The measures taken to reduce the drift to a workable value of about 1 milliampere in the plate current of the power tube in 5 to 10 seconds, corresponding to a voltage change of about 0.001 volt in 5 to 10 seconds at the first grid, were as follows:

(a) To insulate all C batteries by means of paraffin blocks.

(b) To use a somewhat smaller plate voltage on the tubes than recommended for ordinary radio use.[3]

[3] It should be noted that the so-called plate voltage specified by some manufacturers really denotes the B-battery voltage to be applied to a plate circuit in which a specified external resistance is placed.

(c) To use storage cells for B batteries.

(d) To use 50,000-ohm coupling resistances.

In spite of these precautions it occasionally happens on days of high humidity that the amplifier is unworkable because of a large drift. Trouble from this source is greatly reduced when the amplification is reduced.

A second serious difficulty arose in the attempt to use a common B battery for all stages. It will be seen that the B battery is common to the input circuits in this type of amplifier and forms a coupling between the stages. If the B battery remained of constant voltage, this coupling would cause no trouble, but unfortunately the voltage depends on the current drain. The variation is especially large for the ordinary light-duty dry cells which we attempted to use at first, and many peculiar effects were found. This difficulty was removed by the use of storage B batteries and by using a separate battery for the power tube.

It is found that the amplification changes with time. It is necessary to run the amplifier about 15 minutes before beginning observations and to measure the amplification frequently. To insure that the tubes are always worked on the linear portion of the characteristic curve milliammeters are included in each stage. The characteristics of the amplifier are such that blocking does not occur if the limits of the power tube are not exceeded. It is necessary occasionally to check the characteristics of each tube.

Since in various applications it was desired to measure variations from 0.005 volt to 0.10 volt, or even 0.2 volt, it was desirable to be able to vary the amplification in reasonably close steps. Each stage amplifies about 12 to 15 times, and to secure finer adjustment than was possible by varying the number of stages arrangements were made in the first stage to pass on one-fourth, one-half, three-fourths, or all of the drop across the coupling resistance.

A direct check was made of the nonselectivity of the amplifier (compensating circuit omitted) by observing the amplification of a known alternating current and comparing with the direct current amplification. Frequencies up to 120 cycles per second were tried and no selectivity was observed.

OUTPUT CIRCUIT

In the output circuit the potential difference across a fixed resistance is balanced by a voltage divider. The tap on the voltage divider resistance is set once for all, so that no current flows in the balancing circuit when the plate current of the power tube is at the value corresponding to the midpoint of the linear part of the characteristic curve. Balancing is subsequently made by the C bias voltage divider of one of the stages of the amplifier. Under these conditions the relation between the unbalanced current shown on the milliammeter and the voltage applied to the grid of the first tube is linear over a considerable range (± 10 milliamperes). The alternating current still remaining is measured by an alternating current milliammeter of range 0 to 5 milliamperes. If the alternating current does not exceed 5 milliammeters, and the mean values of the plate currents of the various stages are at the proper values, the tubes are known to be working within the linear range of the characteristic curves and are not blocking.

COMPENSATING CIRCUIT

It has been shown that compensation can be made for the lag of the wire by passing on at some stage the voltage variation in a circuit containing inductance L, and resistance R, such that L/R equals the time constant M, of the wire. We may compute the performance of circuits containing a vacuum tube by assuming (reference 18) that the tube impresses a voltage μe, μ being the amplification factor, and e the grid voltage, in a circuit consisting of the external impedance and the plate resistance of the tube R_o. The distortion term is negligible if the tube is operated in the linear range of the characteristic curve. Since the grid-filament resistance of the succeeding tube is very high and the grid current negligibly small, we need only to compute the voltage variations passed on to the next tube.

The compensating circuit (fig. 5b) consists of a tube of plate resistance R_o, a resistance R_1, and an inductance L of resistance R_2. The voltage drop across the inductance L and resistance R_2 is passed on to the next stage. As is customary in similar calculations we consider only the variable part of the plate current which we call J_o, and the variable part of the grid voltage, which we call e. We have then

$$\mu e = J_o\,(R_o+R_1+R_2+ipL)$$

and the voltage passed on to the next tube as

$$J_o\,(R_2+ipL)$$

or

$$\frac{\mu e\,(R_2+ipL)}{R_o+R_1+R_2+ipL}$$

After some reduction, we find this expression equivalent to

$$\frac{\mu e\sqrt{1+\frac{p^2L^2}{R_2^2}}}{\sqrt{\left(\frac{R_o+R_1}{R_2}+1\right)^2+\frac{p^2L^2}{R_2^2}}}\exp.\left[i\tan^{-1}\frac{\frac{pL}{R_2}}{1+\frac{R_2}{R_o+R_1}+\frac{p^2L^2}{R_2(R_o+R_1)}}\right]$$

If we can meet the three conditions, that $\frac{R_o+R_1}{R_2}+1$ is large as compared with $\frac{pL}{R_2}$, that $\frac{R_2}{R_o+R_1}+\frac{p^2L^2}{R_2(R_o+R_1)}$ is small compared to unity, and that $\frac{L}{R_2}$ is equal to the time constant of the wire, we have the compensation desired.

In the actual circuit, $R_o=50{,}000$ ohms, $R_1=100{,}000$ ohms, R_2 varies from 700 to 5,000 ohms (by means of adjustable rheostat) and $L=13$ henries. Carrying through the limiting cases for a frequency of 100 cycles—i. e., $p=628$—we have the following table:

R_2	$\frac{R_o+R_1}{R_2}$	$\frac{pL}{R_2}$	$\left(\frac{R_o+R_1}{R_2}+1\right)^2$	$\frac{p^2L^2}{R_2^2}$	$\frac{\frac{p^2L^2}{R_2^2}}{\left(\frac{R_o+R_1}{R_2}+1\right)^2}$	$\frac{R_2}{R_o+R_1}+\frac{p^2L^2}{R_2(R_o+R_1)}$
700	215	11.65	46,700	136	0.0029	0.69
5,000	30	1.63	961	2.66	.0027	.12

The error in compensation for amplitude is only 0.15 per cent, but the errors in phase are apparently much greater. We find from the preceding equations that the phase errors could be made smaller by using 1 megohm for R_1, but this procedure would reduce the amplification, and since the mean square value does not depend on the relative phases of the various components into which the complex wave shape may be resolved, we have actually used 100,000 ohms. Furthermore, the errors are in reality not as great as they seem. Thus for $R_2=700$, the phase is advanced by an amount $\tan^{-1}\frac{11.65}{1.69}=6.89$ instead of the correct amount $\tan^{-1} 11.65$, i. e., 81.7° instead of 85.1°, and for $R_2=5{,}000$, by 55.4° instead of the correct 58.5°. In other words, the error in phase is only 3°. The compensation is therefore regarded as satisfactory for frequencies up to 100 cycles. The error at 500 cycles is only 4 per cent in amplitude and about 15° to 19° in phase.

The compensation is secured at the expense of the voltage amplification of this stage which is

$$\frac{\mu\sqrt{1+\frac{p^2L^2}{R_2^2}}}{1+\frac{R_o+R_1}{R_2}}$$

For the tube used ($\mu=30$) and the values given above, we find that this stage does not amplify at all but actually reduces the voltage under some conditions. Thus the direct-current amplification is 0.14 for $R_2=700$ and 0.59 for $R_2=5{,}000$ and the amplification at 100 cycles 16.4 for $R_2=700$ and 1.1 for $R_2=5{,}000$. The compensation is therefore secured by reducing the low-frequency amplification and an additional stage of amplification is required to make up for this loss.

To secure the computed performance it is very necessary that the inductance L be independent of the frequency and of the current, which means that no iron can be used. The resistance must also be reasonably low. M. Brooks and H. M. Turner (reference 19) have shown that to secure a given value of L/R, a definite weight of wire is required, and in the present case it was only necessary to wind No. 26 wire (about 14 pounds) on a spool until the desired L/R was obtained, in this case about 0.018 second, L being 13 henries and R 735 ohms. Smaller values of L/R could then be secured by introducing additional resistance in series with the coil.

EXPERIMENTAL TEST OF COMPENSATING CIRCUIT

Because of the importance of this feature of our apparatus, we made special tests to obtain a check on the accuracy of compensation. The experimental arrangement for this purpose (shown in fig. 6) consisted of a motor-driven crank, connecting rod, and lever system for imparting an approximately harmonic motion to a slide operating on the outside of a brass pipe 1¼ inches in diameter. The pipe was provided with a bell-mouth entrance at one end, and the other end was inserted in the wall of the 54-inch wind tunnel. A hot wire could be mounted within the pipe on the slide through a narrow slot.

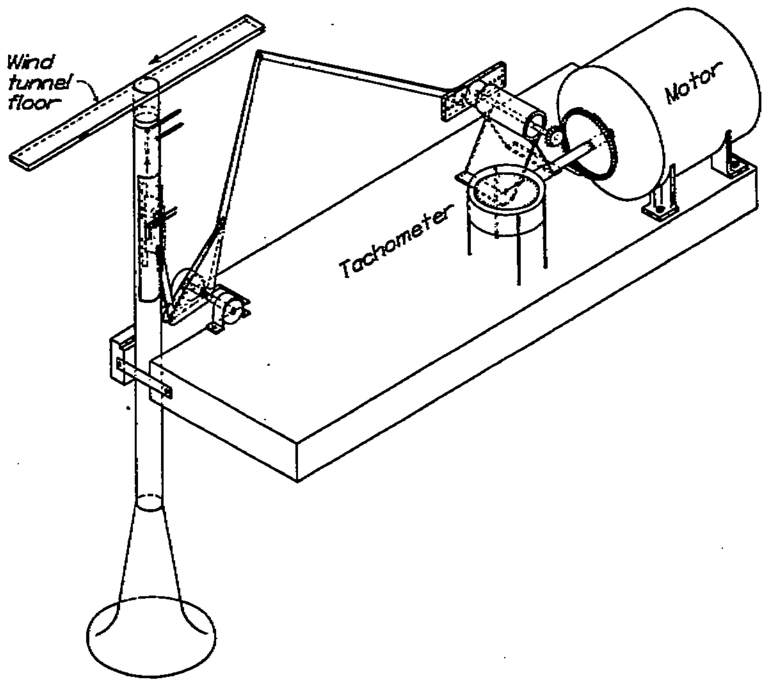


FIGURE 6.—Apparatus for experimental test of compensation

The object of this arrangement was to be able to maintain a reasonably steady air stream past the wire and to superpose on this steady relative motion a harmonic motion by oscillating the wire. The experiment was made in a tube rather than in the wind tunnel itself, so as to be able to eliminate all interference from the motor, crank, connecting rod, and lever systems.

The procedure was as follows: The hot-wire mounting was removed from the pipe and a relation found between the speed at the center of the pipe and the speed indicated by the wind-tunnel gauge. The wire was then replaced and calibrated—i. e., the voltage drop across the wire was measured at several wind speeds. The wire was then oscillated at several frequencies and amplitude so chosen that the product of frequency and amplitude, and therefore the maximum speed produced by the oscillation, was constant, and the reading of the alternating-current milliammeter in the output circuit noted. A typical experiment of this kind gave the following results, the compensating circuit operating:

Mean speed	Double amplitude of motion of wire	Frequency cycles per second	Voltage variation [1]	Corresponding speed variation
Ft./sec.	mm			Ft./sec.
46.9	12.71	20	0.0171	6.3
45.1	6.36	40	.0173	6.1
45.1	4.23	60	.0176	6.3
44.0	0	0	.0056	2.0* (*Due to turbulence of stream in the pipe).

[1] The voltage variations are derived from the milliammeter readings by assuming a sinusoidal wave form, hence multiplying by $2\sqrt{2}$ to get the double amplitude and then multiplying by the amplification ratio. Since the mean speed varies a little from one reading to another, a given voltage variation does not correspond always to exactly the same speed variation.

We may attempt to correct for the unavoidable turbulence in the pipe by assuming that the value for a stream free from turbulence is $\sqrt{6.2^2-2^2}=5.9$ ft./sec. The value computed from the amplitude and frequency, assuming sinusoidal motion, is $2\pi\times20\times\frac{12.71}{305}$, or 5.2 ft./sec. Considering the several sources of error and the difficulties introduced by the turbulence of the stream, the difference between the two values may not be considered excessive.

Further experiments were made by the use of the oscillograph instead of the milliammeter with the hope of being able to average out the effects of turbulence in the pipe. Sample records are given in Figure 7, and a typical set of results in the table. The wire used was not the same as the one used with the milliammeter and the mean air speed was somewhat different, so that the relation between voltage variations and speed variations is not the same as for the preceding table. The value of the time constant was computed as 0.00415 second.

[Uncompensated, 3.25 mm on film=0.004 volt]

Double amplitude of motion of wire	Frequency cycles per second	Mean double amplitude on film	$\sqrt{1+p^2M^2}$ times observed amplitude
		mm	mm
12.71	20	9.6	10.8
6.36	40	6.8	9.8
4.23	60	5.5	10.2
			Mean 10.3 or 0.0127 volt or a speed variation of 5.5 ft./sec.

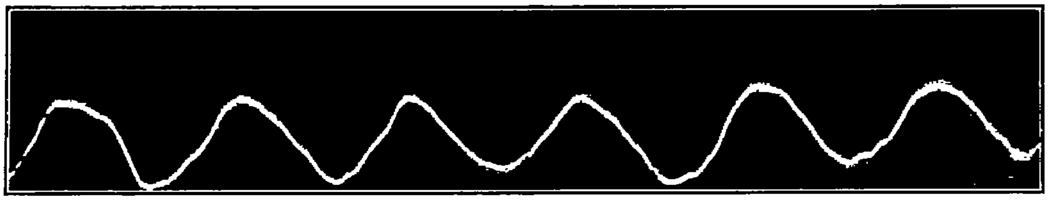
[Compensated, 1 mm on film=0.004 volt]

Double amplitude of motion of wire	Frequency	Mean double amplitude on film
mm	Cycles/sec.	mm
12.71	20	3.1
6.36	40	3.0
4.23	60	3.0
		Mean 3.03 or 0.0121 volt or 5.3 ft./sec.

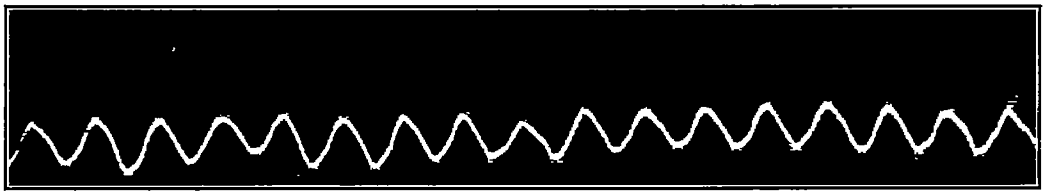
These values show better agreement with the computed value of 5.2 ft./sec. and illustrate the marked difference in response when uncompensated as compared with compensated.

As a result of these and other measurements, we believe that the net effect of the many sources of error, including the errors in determining the time constant of the wire, do not on the average exceed 10 per cent, and since we are attempting to measure variations of air speed which do not remain of constant amplitude this error is not excessive.

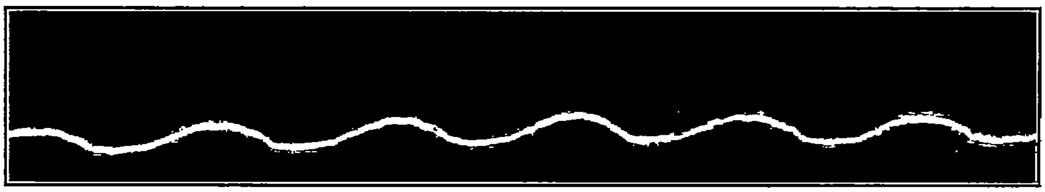
We appreciate full well the errors involved in using a wire 7 mm long in a pipe 32 mm in diameter because of the variation of air speed over different parts of the wire, but we feel that a direct mounting in the wind tunnel involves very much larger errors due to interference effects.



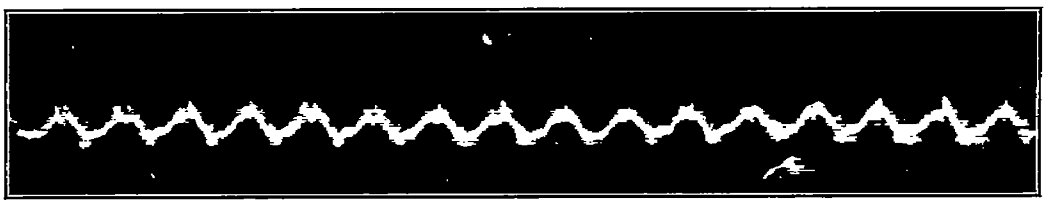
Uncompensated 20 cycles per second



Uncompensated 60 cycles per second



Compensated 20 cycles per second



Compensated 60 cycles per second

FIGURE 7.—Oscillograph records of test of compensation

The impossibility of obtaining a stream free from turbulence and the necessity of keeping the superimposed speed fluctuation reasonably small offer insuperable obstacles to an extremely accurate check, but we may accept these experiments as evidence that no large factor has been overlooked in the theory of the compensator.

The determination of the correct value of M under given conditions requires a knowledge of the diameter, density, temperature coefficient of resistance, resistance at room temperature, specific heat, and mean temperature of the wire. Of these quantities, the wire diameter is most sensitive and is the most difficult to determine accurately. We have used a value obtained by the interferometry section by using the wire to form a wedge between two optical flats and counting the number of fringes. The value is probably correct to ± 2 per cent. A check on this value was obtained by measuring the resistance of a known length of wire. The resistance at room temperature was measured by a sensitive bridge method so that the current through the wire was very small. The density was taken as 21.37 g/cm^3, the temperature coefficient as 0.0037, and the specific heat as 0.032. These values should, strictly speaking, be modified according to the wire temperature, but with the accuracy attainable it does not seem worth while to do so at present. At high frequencies the percentage error in the amplitude of the speed fluctuation is equal to the percentage error in M, while at low frequencies it is somewhat less.

A word should perhaps be said about the influence of the temperature distribution along the wire. The distribution may be calculated approximately from the known heat conductivity of the material, the rate at which heat is generated by the electric current, and the heat loss to the air stream. It is found that the arithmetic mean temperature differs from the maximum temperature by 2½ per cent for a maximum temperature of 150° and by 4½ per cent for a maximum temperature of 500°. The errors introduced in taking various types of mean values do not therefore exceed 2 or 3 per cent. After all, the wire is about 350 diameters long and is not at all short as compared to the diameter.

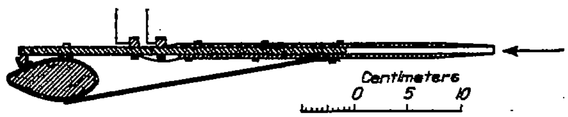


FIGURE 8.—Wire mounting for central part of air stream

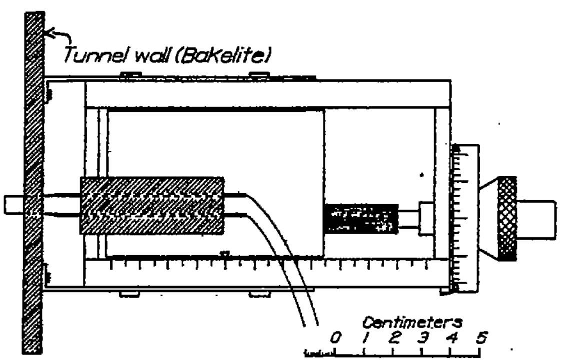


FIGURE 9.—Wire mounting for use near the tunnel wall

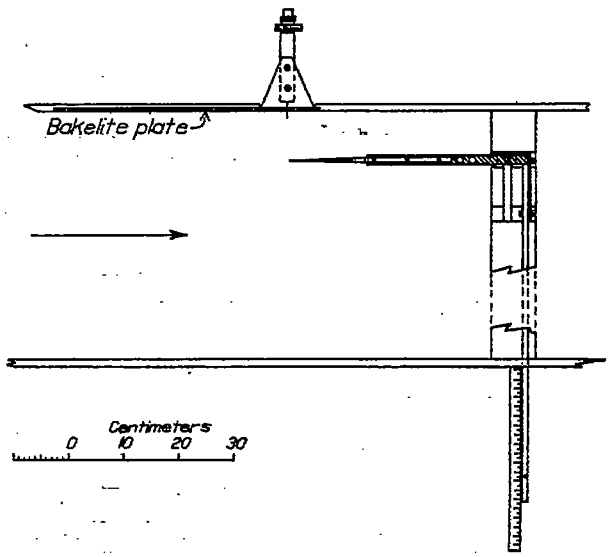


FIGURE 10.—General relation of the two wire mountings

MEASUREMENTS OF WIND TUNNEL TURBULENCE

To illustrate the use of the apparatus described in this paper and the general nature of the results obtainable, we will describe some measurements made in connection with the problem of turbulence in wind tunnels. The methods of mounting the wire and of traversing across the tunnel at a given section are illustrated in Figures 8, 9, and 10, showing the mountings used in the main body of the stream and near the wall. Every effort was made to eliminate interference effects from the supports and to secure reproducible conditions near the wall. A bakelite plate 7.5 cm wide was set into the wall extending about 43 cm upstream and 7 cm downstream from the section at which the traverse was taken. The wire, about 6½ mm. long, was supported by two stiff prongs extending through small holes in the bakelite to a block traveling on a micrometer screw. The zero position of the screw was obtained by observing the reflection of the wire in the smooth bakelite surface. This wire mounting could be used for distances from the wall up to 5 mm. The remainder of the cross section was explored by a wire mounted on two prongs parallel to the wind stream attached to a carriage sliding on a strut about 37 cm downstream. (Figs. 8 and 10.)

The wire was placed in every case at right angles to the wind direction and to the line of traverse. In this position it is least sensitive to directional changes and responds to any change in the resultant speed.

A brief description of the 54-inch wind tunnel in which this work was carried out has been given elsewhere. (Reference 20.) The only change of importance since that description was published has been the removal of the diffuser and the installation of a room honeycomb. Figure 11 shows a sketch of the tunnel with the sections indicated at which traverses were made. Traverses were made only along a single horizontal line toward one wall of the tunnel.

The procedure was as follows: The wire was placed at the center of the tunnel and calibrated against a static plate, which in turn had been calibrated against a Pitot tube placed at the point subsequently occupied by the wire. For the traverse near the wall the wall instrument was calibrated against the center instrument. The tunnel was then operated at the speed at which it was desired to make the traverse and the mean voltage drop across the wire balanced with the wire in the desired traverse position. The time constant of the wire was then computed and the compensating resistance set so that the ratio of the inductance to the resistance was equal to the time constant. The fluctuations were then impressed on the amplifier, the direct-current milliammeter reading of the output balancing circuit reduced to as near zero as possible, and the reading of the alternating-current milliammeter noted. In this process two observers cooperated, and an attempt was made to obtain the best average reading of the alternating-current milliammeter when the direct-current instrument was fluctuating equally about the zero mark. In other words, an effort was made to confine attention to disturbances

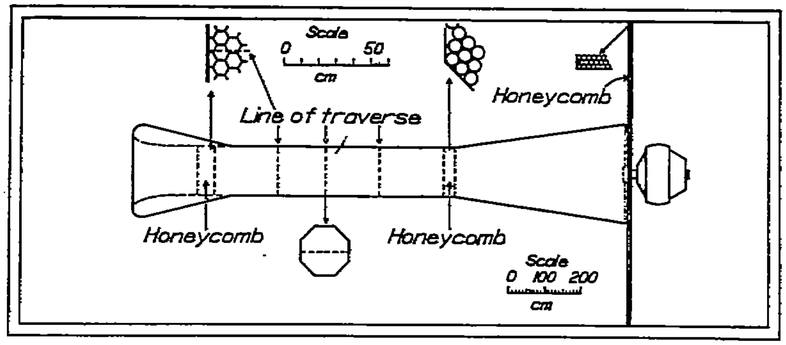


FIGURE 11.—Sketch of 54-inch wind tunnel with traverse positions

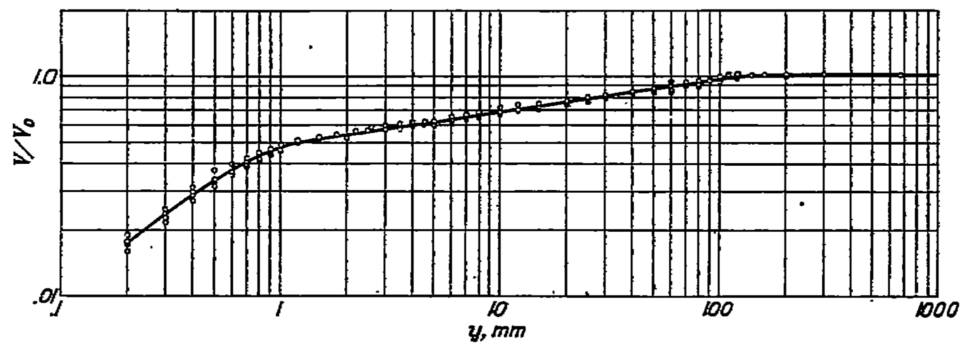


FIGURE 12.—Logarithmic plot of the distribution of mean speed, 5 feet downstream from the working section. y=distance from wall, mm. V=mean speed at distance y. V_0=mean speed at center of working section

of frequency greater than one cycle in one or two seconds. After the reading was obtained the wire was switched out as explained in a previous section and the change in current in the balancing circuit for a known change of voltage determined. The wire was then moved to a new position and the process repeated, the amplification being adjusted if necessary to give a reading in the neighborhood of 2 to 4 milliamperes.

The variations in speed were computed in the following way: We know that the reading of the alternating-current milliammeter is proportional to the square root of the mean square deviation of the resistance from its mean value. We therefore computed the amplitude of the sine curve, giving the same square root of the mean square value and read off from the calibration curve the speeds corresponding to the mean resistance plus this amplitude and the mean resistance minus this amplitude. We called the mean of the two speeds, the mean speed, and their difference the double amplitude of the speed variation.

If the resistance-speed curve were linear, the above procedure would give accurately the speed variation giving the correct square root of the mean square value, irrespective of the actual wave form. The curvature of the resistance-speed curve introduces an error which we see no way of eliminating at present. For small variations the calibration curve may be considered approximately linear, and the errors from this cause are probably not very great.

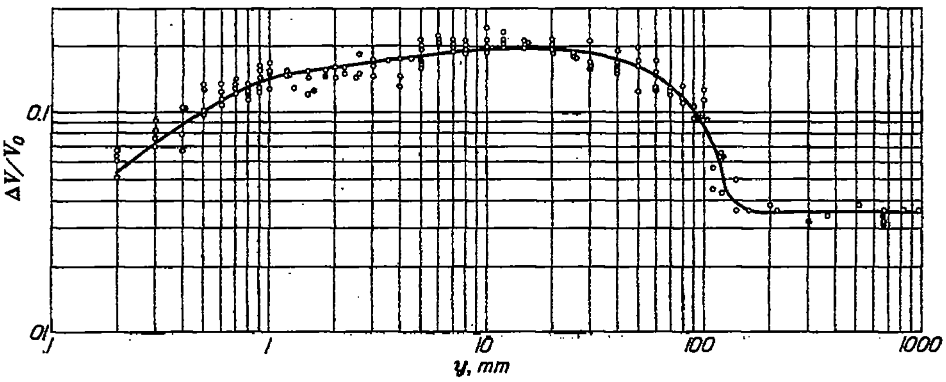


FIGURE 13.—Logarithmic plot of the distribution of the double amplitude of the speed fluctuations, expressed as ratios to the mean speed at the center of the working section. y=distance from wall, mm. ΔV=double amplitude of speed fluctuation. V_o=mean speed at center of working section.

Figure 12 shows the distribution of mean speed along a line from the center of the tunnel to the east wall 5 feet downstream from the working section, plotted with logarithmic scales to show the power law variation. The break in the curve at a distance less than 1 mm is not due to the presence of the wall, since the presence of the wall causes a greater heat loss, cools the wire, and hence gives too high a value for the speed. A method of correction for the effect of the wall has been given by B. J. Van der Hegge Zijnen (reference 21), in which it is assumed that the heat loss to the wall is independent of the wind velocity. This correction has been applied to our values. The correction was negligible for distances greater than 1.2 mm. Van der Hegge

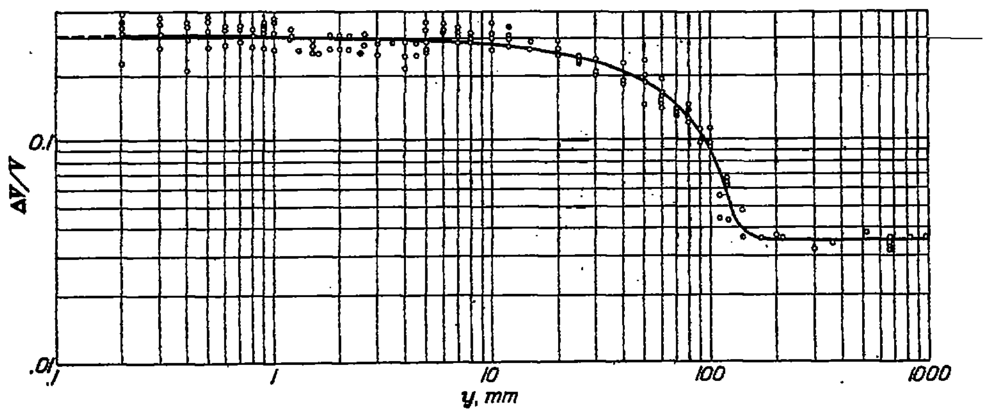


FIGURE 14.—Logarithmic plot of the distribution of the double amplitude of the speed fluctuations, expressed as ratios to the local mean speed. y=distance from wall, mm. ΔV=double amplitude of speed fluctuation. V=mean speed at distance y

Zijnen gives an interpretation of this peculiarity of the distribution curve, for which reference may be made to the original paper.

It is seen that the air stream consists of a core of approximately uniform speed surrounded by a region near the wall in which the speed decreases. The equation of the curve of mean speed in this "boundary layer" is $\frac{V}{V_o}=\left(\frac{y}{\delta}\right)^n$ where V is the local mean speed, V_o the speed of the core at the working section, y the distance from the wall, δ, the thickness of the boundary layer, in this case 135 mm, and n a constant, in this case 0.15.

The accuracy of measurement of the mean speed is not as great as one might secure by the use of a wire of greater diameter. The wire used, 0.017 mm in diameter, was extremely fragile and broke very frequently. The calibration curve was subject to erratic changes of the same type as those described by Bailey and Simmons (reference 22) for a wire 0.025 mm in diameter

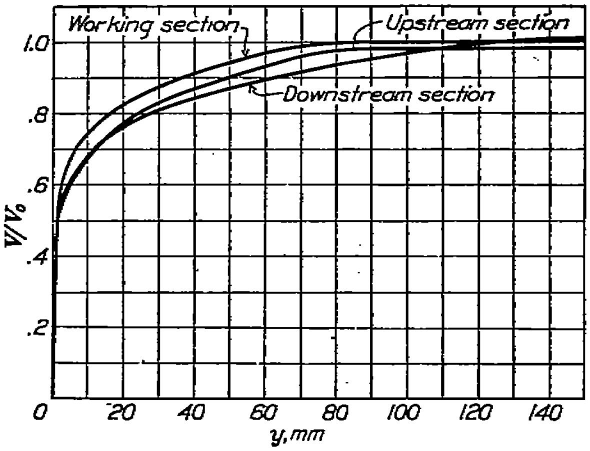


FIGURE 15.—Distribution of mean speed at three sections. y=distance from wall, mm. V=mean speed at distance y. V_0=mean speed at center of working section

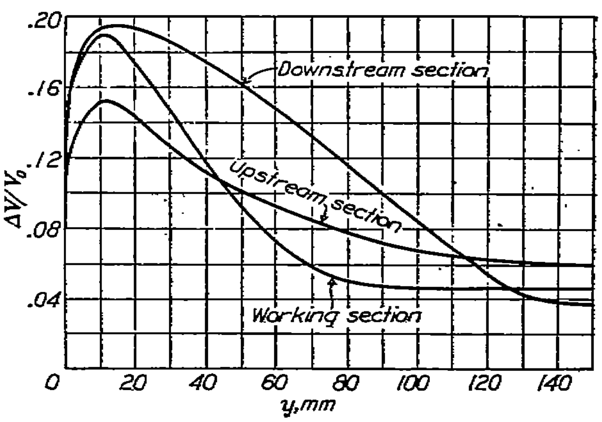


FIGURE 16.—Distribution of the double amplitude of the speed fluctuations at three sections, expressed as ratios to the mean speed at the center of the working section. y=distance from wall, mm. ΔV=double amplitude of speed fluctuation. V_0=mean speed at center of working section

but even more troublesome. In spite of calibrations two and four times per day and of numerous returns to a reference point during a traverse, it was difficult to secure an accuracy of better than ±1.5 per cent in the mean speed. Some of the difficulty arose from the ability to see fluctuations on the balancing instrument. To measure fluctuations, the wire must be small, and judging by the frequency of failure the diameter used is close to the lower limit of mechanical strength for use at speeds up to 60 foot-seconds. The traverses were made at a mean tunnel speed of approximately 55 foot-seconds.

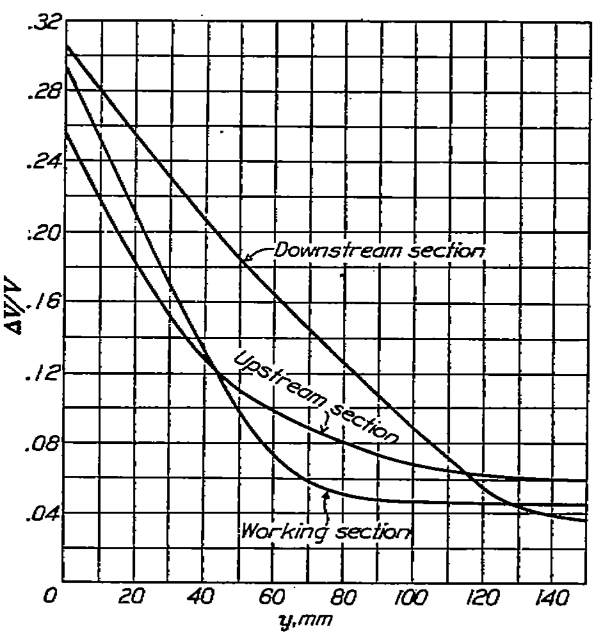


FIGURE 17.—Distribution of the double amplitude of the speed fluctuations at three sections, expressed as ratios to the local mean speed. y=distance from wall, mm. ΔV=double amplitude of speed fluctuation. V=mean speed at distance y

Figure 13 shows the distribution of speed fluctuations, also on a logarithmic scale. The ordinates of the curve are equal to the double amplitudes of the speed fluctuations expressed as a ratio to the mean speed of the central core of the air stream at the working section, derived from the square root of the mean square values on the assumption of a sinusoidal wave form as previously explained. The abscissas are distances from the wall. There is a definite correlation between the variation of the speed fluctuations and the variation of mean speed. Over the central core the double amplitude of the fluctuations of speed is reasonably constant, as is the mean speed. In the boundary layer the fluctuations increase to a flat maximum at about 15 mm from the wall and then decrease as the wall is approached.

In Figure 14 the speed fluctuations are expressed as ratios to the mean *local speed* instead of to the speed of the central core as in Figure 13. It is probably only a coincidence that the limiting value of the amplitude (i. e., half the double amplitude) of the speed fluctuation at the wall expressed in this way (0.15) is nearly equal to the exponent occurring in the law of distribution of the mean speed (0.15).

In Figures 15, 16, and 17 we have plotted on a linear scale the average curves for the working section, a section 150 cm downstream, and a section 131 cm upstream, the locations being shown on Figure 11. Comparison of the working section and the downstream section shows the expected course of events. The boundary layer increases in thickness, and consequently the speed of the core increases a little. The large variations of speed extend to the limit of the new boundary layer, while the variations of speed in the core are somewhat reduced.

The upstream traverse shows anomalous results, repeatedly checked, which we believe to be due to the fact that the honeycomb axis is at a small angle to the tunnel axis. The traverses shown cover a comparatively limited region and can not be considered as representative of conditions over the entire cross section. Since we do not have sufficient data to make a comprehensive report on the speed variations in our 54-inch wind tunnel and the values given are intended as illustrative of the kind of results obtainable, we shall omit any further discussion.

The precision or perhaps lack of precision is indicated clearly in Figure 13. We believe that the chief difficulty lies in the fact that there are actual changes in the mean values for the time intervals used—i. e., the turbulence as defined by our method of measurement is not constant.

FIGURE 18.—Oscillograph record of speed fluctuation near the wall

We have, of course, been most interested in the turbulence in the core of the air stream. A typical set of values of the double amplitude of the speed variation taken in the core at the working section is shown below.

Distance from center (cm)	First run	Second run
46 west	0.043	0.041
30 west	.045	.040
15 west	.041	.041
0	.043	.041
15 east	.045	.041
30 east	.047	.043
46 east	.045	.043
Mean	.044	.041

The mean of a number of readings at the three sections gave the following results:

131 cm upstream	0.060
Working section	.043
150 cm downstream	.035

As a matter of interest, an oscillograph record of the wave form near the wall is shown in Figure 18. The light ray is interrupted at intervals of approximately 0.15 second.

MEASUREMENTS BEHIND CYLINDERS

In the development of the apparatus we made a few measurements of the fluctuations present behind a cylinder 3 inches (7.5 cm) in diameter and extending completely across the tunnel at the working section. We wish to use these measurements (figs. 19 and 20) to illustrate another application of the apparatus. While the accuracy of these measurements is not very great and the cylinder was much too large for the tunnel, blocking about 7 per cent of the cross-sectional area, the qualitative information as to the rapidity with which the Karman vortex system is dissipated is of some interest. The diagrams are self-explanatory.

CONCLUSION

The apparatus described is very bulky, far from portable, and in many respects inconvenient to use. We believe, however, that a great improvement is possible, and we are now engaged in redesigning the amplifier, using very large coupling condensers in place of the large C batteries. By sacrificing a little in sensitivity and accuracy the wire may be heated from a 6 or 12 volt storage battery.[4] With several changes of this nature we hope to produce a more portable apparatus. The extension to measurements of directional variations using the 2-wire arrangement of Burgers is relatively easy.

In conclusion we wish to acknowledge with appreciation the assistance of our associates, Dr. Arthur E. Ruark, Messrs. K. H. Simpson, P. S. Balliff, B. H. Monish, and W. H. Boyd.

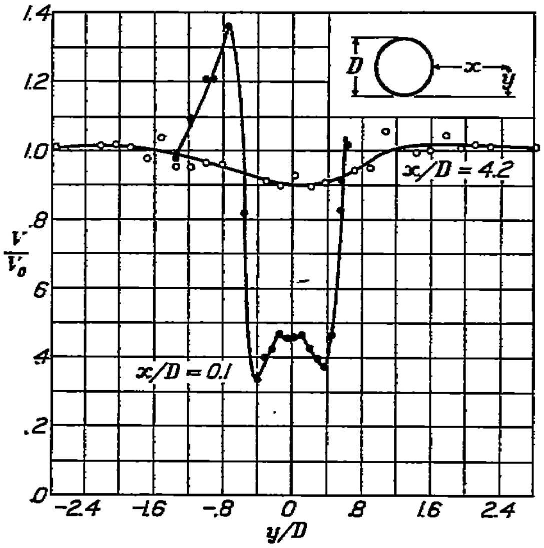


FIGURE 19.—Distribution of mean speed behind a cylinder 3 inches in diameter. V=mean speed at point (x, y). V_0=mean speed of undisturbed stream

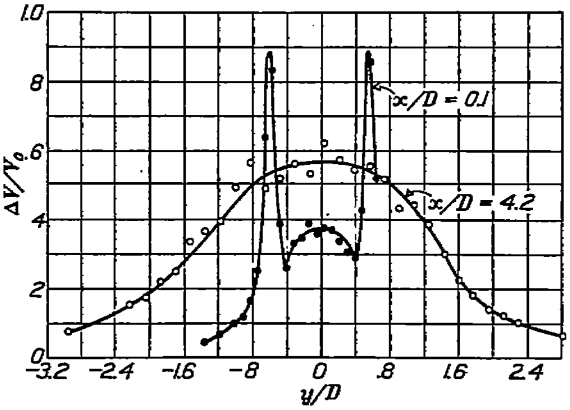


FIGURE 20.—Distribution of double amplitude of speed fluctuations behind a cylinder 3 inches in diameter. ΔV=double amplitude of speed fluctuations at point (x, y). V_0=mean speed of undisturbed air stream. Cf. Figure 19 for coordinates (x, y)

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[4] As this paper goes to press, the redesigned equipment has been in use for several months and direct correlation has been obtained between measurements of forces on spheres and streamline models and measurements of speed fluctuations.

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